The first term on the RHS of the previous equation is

$$\int_{-\infty}^{\infty} \mathrm{d}z \int_{-\infty}^{\infty} \omega \frac{\partial \phi}{\partial y} \, \mathrm{d}y = 0$$

$$\int_{-\infty}^{\infty} y \, \mathrm{d}y \int_{-\infty}^{\infty} \omega \frac{\partial \phi}{\partial z} \, \mathrm{d}z = \int_{-b}^{b} \omega y(\phi_1 - \phi_u) \, \mathrm{d}y$$

The second term in Eq. (15) is E, the kinetic energy of the flow per unit distance at the Trefftz plane, and is shown to be

$$E = -\rho \int_{0}^{b} \omega y(\phi_u - \phi_l) \, \mathrm{d}y$$

Hence, the expression for the suction force given by Eq. (15) simplifies to

$$F_x = \rho \int_{0}^{b} \omega y(\phi_u - \phi_l) \, \mathrm{d}y = -\frac{\pi \rho \omega^2 b^4}{16}$$

It can be verified that the same expression for the suction force is obtained from the conservation of energy.

Discussion and Conclusions

From the foregoing analysis it is seen that the expressions obtained for the normal and suction forces are exact. These results do not depend on the planform shape. In particular the circulation distribution is elliptic and does not depend on the shape of the leading edge. It is worthwhile mentioning that the same results were obtained by Jones in his theory of low aspect ratio wings by treating the crossflow past a triangular wing to be approximately two dimensional. For the cases considered in the present work, the aspect ratio of the wing is zero, and hence, this analysis gives the leading term in an asymptotic expansion for small aspect ratio wings.

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Effect of Angle of Attack on Roll Characteristics of 65-Degree Delta Wing

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Introduction

FOR aircraft with highly swept wing leading edges the high-alpha aerodynamics are dominated by the effects of vortex breakdown. This Note describes the physical flow processes causing the observed large effect of angle of attack on the roll–trim characteristics of a 65-deg delta wing.

The experimental results[1] in Fig. 1 for a 65-deg delta wing show that when the inclination σ of the roll axis was increased from 30 to 35 deg the delta wing lost its stable trim point at zero roll angle. The multiple roll–trim characteristics for σ = 30 deg have been analyzed extensively,[2–7] whereas very little has been published for other σ values. In the case of σ = 30 deg, the rapid aft movement of vortex breakdown on the leeward wing-half[1] (Fig. 2a), when represented by a discontinuous, jumpwise movement,[5–7] resulted in the idealized characteristics shown in Fig. 3a. The measured $C_l(\phi)$ characteristics, although being highly nonlinear, are continuous rather than discontinuous, with the statically destabilizing data trend starting at $|\phi| = \phi_{cr}$. Figure 2a shows that when $|\phi| > \phi_{cr} \approx 4$ deg, vortex breakdown moves very rapidly toward the trailing edge on the leeward, rising wing-half. The associated statically destabilizing increase of the vortex-induced lift overpowers the statically stabilizing data trend generated by the vortex forward of vortex breakdown,[8] resulting in the measured statically destabilizing $C_l(\phi)$ trend in Fig. 3a for $\phi_{cr} < |\phi| < \phi_d$.

On the right, windward wing-half the vortex breakdown moves steadily forward toward the apex with increasing roll angle because of the associated decrease of the effective leading-edge sweep. The breakdown reaches the apex when $|\phi|$ approaches 14 deg (Fig. 2a). Recent experimental results for a 60-deg delta-wing-body configuration[9] have revealed that a dramatic change of the vortex-induced loads would occur

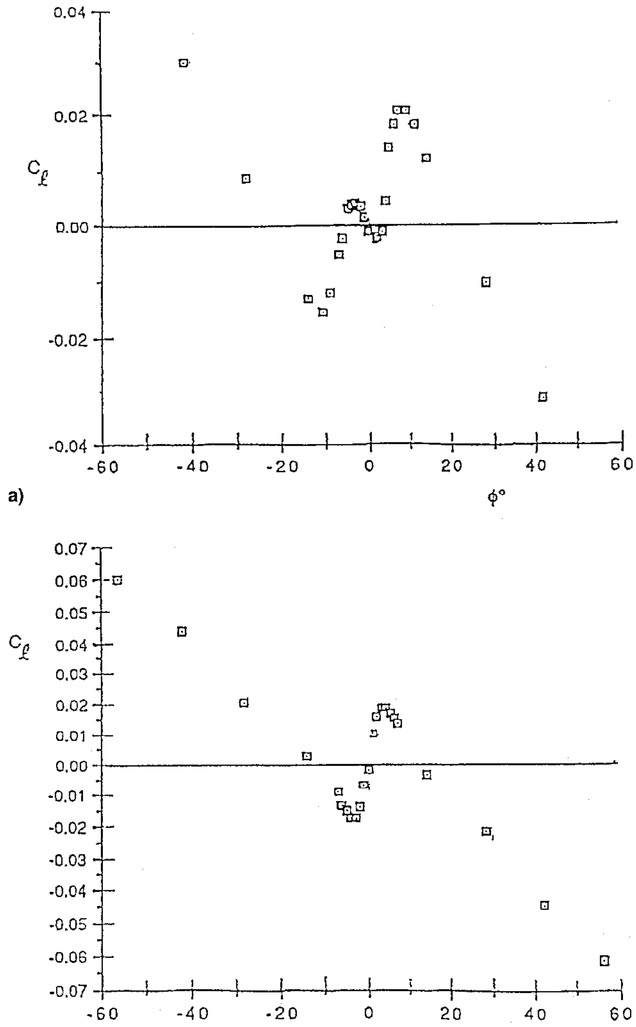

Fig. 1 $C_l(\phi)$ characteristics of 65-deg delta wing.[1] σ = a) 30 and b) 35 deg.

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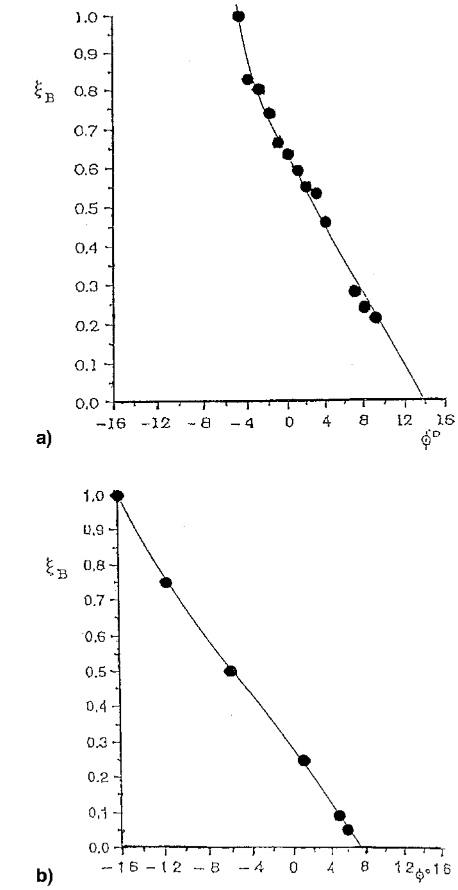

Fig. 2 Effect of roll angle on vortex breakdown of 65-deg delta wing.[1] σ = a) 30 and b) 35 deg.

when the breakdown of the windward vortex reaches the apex, as the moderate suction generated by the leading-edge vortex downstream of a spiral type of vortex breakdown is wiped out. That is, the spiral leading-edge vortex is spilled downstream much as the leading-edge vortex in the case of dynamic airfoil stall.[10] For the 65-deg delta wing in Fig. 3a this statically destabilizing effect from the windward wing-half could account for the experimental overshoot of the idealized characteristics at $\phi_d < \phi < \phi_s$, where $\phi_s \approx 14$ deg. At $|\phi| > \phi_s \approx 14$ deg, the windward wing-half is completely stalled, contributing insignificantly to the rolling moment; whereas the leeward wing-half generates the full, statically stabilizing, vortex-induced loading,[8] resulting in a stable trim point at $\phi \approx 20$ deg (Fig. 3a).

For roll perturbations around $\Phi = 0$ at $\sigma = 30$ deg, where vortex breakdown occurs at 60% chord[1] (Fig. 2a), the stabilizing vortex-induced loads forward of vortex breakdown dominate over the destabilizing effect of the roll-induced movement of the breakdown location (Fig. 3a). This produced the stable trim point at $\Phi = 0$. When the magnitude of the roll-induced perturbation is increased to $|\phi| > \phi_{cr}$; however, the breakdown movement starts to dominate, generating the highly nonlinear, statically destabilizing data trend in Fig. 3a.

For $\sigma = 35$ deg, the $C_l(\phi)$ characteristics take the shape shown in Fig. 3b. At $\phi = 0$, where vortex breakdown is located at 25% chord (Fig. 2b), the statically stabilizing effect of the vortex-induced lift forward of breakdown is completely insignificant compared to the statically destabilizing effect of the movement of vortex breakdown. This results in the very steep, positive $C_l(\phi)$ slope at $\phi = 0$. The overshoot of the idealized, linear characteristics at $|\phi| > 3$ deg could be caused by the statically destabilizing effect of the movement of vortex breakdown toward the apex on the windward, dipping wing-half when $|\phi| = 7$ deg is approached (Fig. 2b). This is a data trend

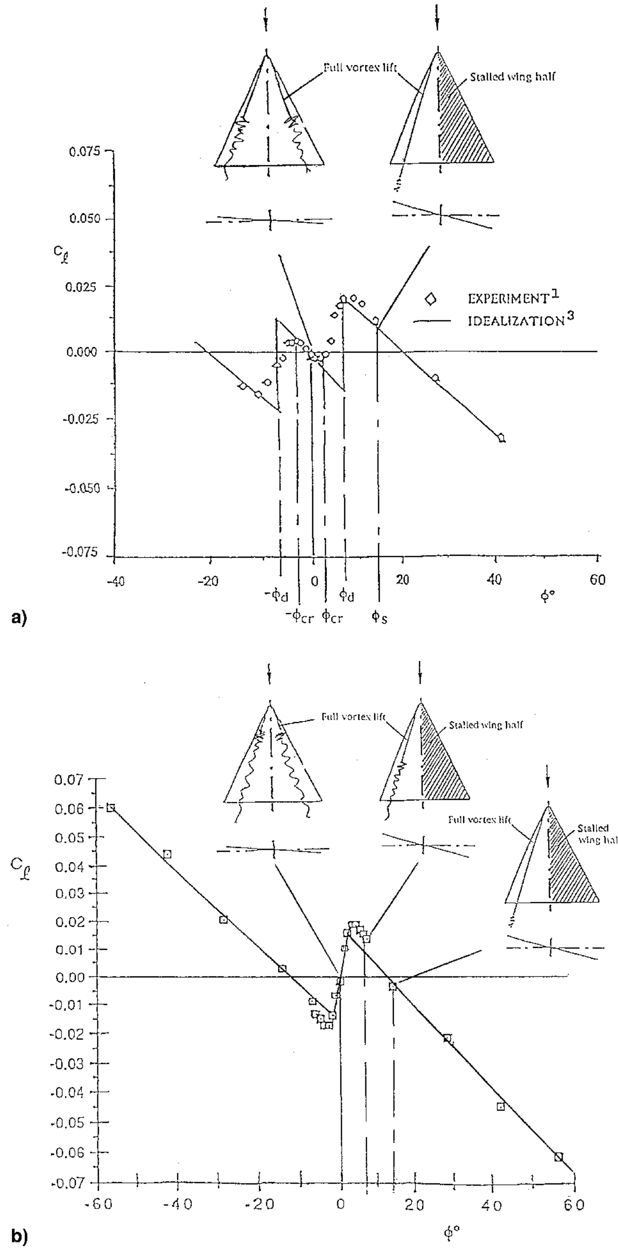

Fig. 3 Evolution of $C_l(\phi)$ characteristics of 65-deg delta wing. σ = a) 30 and b) 35 deg.

augmented by the moderate speeding-up of the aft movement of vortex breakdown on the leeward, rising wing-half until breakdown reaches the trailing edge at $|\phi| = 16$ deg.

As the contributions from leeward and windward wing-halves are associated with different time–history effects,[7] it is important to be able to identify them when determining the dynamic loads on an agile aircraft performing maneuvers at high angles of attack. The flow sketches in Figs. 3a and 3b show the model geometry to be a pure delta wing. It should be emphasized, however, that the flow physics for the tested model are very different from those for a pure delta wing because of the effect of the fuselage.[11]

Conclusions

The big difference between the $C_l(\phi)$ characteristics at σ = 30 and 35 deg is a direct consequence of where vortex breakdown occurs at $\phi = 0$: 60% chord for $\sigma = 30$ deg compared to 25% chord for $\sigma = 35$ deg. Thus, in the former case the vortex-induced lift forward of breakdown dominates, generating a stable trim point at $\phi = 0$; whereas in the latter case the movement of vortex breakdown creates the dominant loads, generating a statically unstable $C_l(\phi)$ slope at $\phi = 0$. In both cases the highly nonlinear $C_l(\phi)$ characteristics away from $\phi = 0$ are generated by two critical flow states, i.e., vortex breakdown leaving the leeward wing-half and reaching the apex on the windward wing-half, respectively. It should be emphasized that the aerodynamic characteristics discussed here do not represent those of a pure 65-deg delta wing because of the camber effect induced by the upwash from the centerbody.

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